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Spacecraft Attitude Acquisition from an Arbitrary Spinning or Tumbling State

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A simple attitude acquisition technique has been developed for use on a spacecraft containing a symmetric rotor. Through this method, the spin axis of the rotor may be aligned parallel to the total angular momentum vector of the spacecraft. The maneuver may be performed regardless of the orientation of the rotor within the spacecraft, and regardless of the initial rotational state of the spacecraft. Alignment of the rotor axis with the angular momentum vector is achieved by first spinning up the rotor, and then allowing passive energy dissipation mechanisms to act while the rotor is made to spin at a constant rate relative to the remainder of the spacecraft. It is demonstrated that, with the proper choice of rotor speed, the initial conditions and dynamic history of the spacecraft during rotor spinup are irrelevant to the ultimate convergence of the rotor axis to the desired orientation. Because of this, a number of maneuvers previously thought to be impossible are now known to be feasible.

Introduction

SEVERAL papers[1-3] and a text[4] have described and analyzed an attitude maneuver that is sometimes known as the "dual-spin turn." The dual-spin turn is an open-loop procedure for acquiring a stable dual-spin configuration when the rotor axis is parallel to the spacecraft's minimum or intermediate moment of inertia axis, and the spacecraft is initially in a state of simple spin about the axis of maximum moment of inertia. Reorientation is accomplished by using a motor to torque the rotor until it reaches a speed consistent with the stability requirements for the nominal operational orientation. Reaction torque applied to the spacecraft body during rotor spinup produces most of the required attitude change. Perfect alignment of the rotor axis with the angular momentum vector, however, cannot be achieved by open-loop torquing alone. A residual nutation will essentially always be present. Since the dual-spin configuration is stable, however, ultimate convergence can be assured by incorporating a passive energy dissipation mechanism in the platform of the spacecraft. The first spacecraft to employ this attitude acquisition technique was RCA Satcom I in December of 1975.

In analyzing the dual-spin turn, the main thrust of previous investigations has been examination of the dynamic effects of rotor spinup. This work has placed the dual-spin turn on a firm analytical footing, but it has also indicated that other maneuvers of this type may not be feasible. Kaplan[4] has labeled as "unacceptable" one case that is of particular interest to spacecraft designers. In this case, the spacecraft is initially spinning about its axis of minimum moment of inertia and the rotor is aligned with its axis parallel to the spacecraft's axis of maximum moment of inertia. When the rotor is spun up, reaction torques cause the spacecraft to turn so that the rotor momentum vector is oriented opposite to the total angular momentum vector, rather than in the same direction, as is required. Furthermore, rather than despinning, the platform acquires an even higher momentum. To make matters worse, this inverted condition is unstable. The present work, however, demonstrates that this instability can be used to advantage and that this maneuver and others can be satisfactorily completed despite initial divergence from the desired end condition.

The technique presented in this paper is essentially a generalization of the dual-spin turn. Specifically, it is demonstrated that the rotor axis may be aligned with the angular momentum vector regardless of the orientation of the rotor within the spacecraft, regardless of the initial conditions, and regardless of the dynamics associated with rotor spinup. The maneuver is performed by passive energy dissipation that occurs after the rotor is up to speed. The only active system required is the one that maintains the rotor at a constant speed relative to the platform.

The maneuver is based on minimization of the "core" energy, which is defined as being equal to the total energy of the spacecraft minus that portion of the rotor energy that is due to the relative rotation between the rotor and the platform. Hubert[5,6] has shown that, if a dissipative device is mounted on the platform, the core energy of a dual-spin spacecraft will always approach a minimum. Thus, if the rotor speed is such that the desired orientation represents a core energy minimum, and if the minimum is unique, then convergence to that attitude is assured. The current work demonstrates that, if the rotor axis is parallel to a spacecraft principal axis, then a rotor speed can always be found to create the necessary unique minimum. Also treated is the case in which the rotor axis is aligned with a nonprincipal spacecraft axis.

Analysis

The spacecraft treated by this study is illustrated in Fig. 1. It consists of a rigid platform with arbitrary inertial properties and a rigid symmetric rotor. A motor mounted on the platform is used to spin up the rotor and, when the desired speed is achieved, continues to drive the rotor so that it maintains a constant rate relative to the platform. In general, a nonzero torque is required to maintain a constant relative rate during arbitrary spacecraft rotational motion. It is assumed that no external torques are applied to the spacecraft and, therefore, that the total angular momentum h_s is constant both in magnitude and inertial orientation.

Orthogonal coordinate axes, denoted 1, 2, and 3, are fixed in the platform with the origin located at the center of mass. The 3 axis is chosen to be parallel to the rotor axis and the 1 and 2 axes are selected so that the 1-2 product of inertia is identically zero. Eliminating the 1-2 product simplifies the mathematics and results in no loss of generality.

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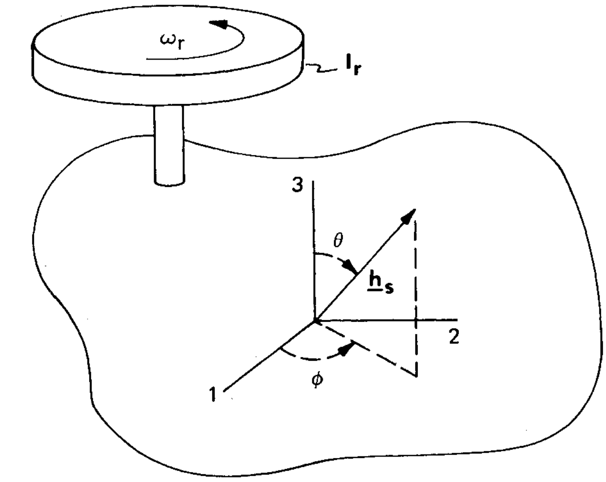

Fig. 1 Spacecraft configuration and coordinate system.

The orientation of the total angular momentum vector relative to the platform is described by the angles θ and ϕ; the angle between the positive 3 axis and $\boldsymbol{h}_s$, and the angle between the positive 1 axis and the projection of $\boldsymbol{h}_s$ on the 1-2 plane. These angles are, in general, functions of time. The angle θ will be referred to as the "nutation angle." This definition is used in a very broad sense and is not meant to imply that the motion is always characterized by nutation as it is understood in classical rigid-body dynamics.

Although it is not illustrated in Fig. 1, the platform is assumed to contain an energy dissipation device. It is also assumed, however, that the mass, inertias, and motions of this device are sufficiently small that its energy is negligible relative to the spacecraft core energy. The dissipator, therefore, will be treated as an undefined "energy-sink" for the purposes of the energy analysis.

It is convenient to be able to express energy relationships and the equations of motion in matrix form. Toward this end, the products of inertia are defined to include the negative sign. In other words, the off-diagonal elements of the inertia matrix are, by our definition, the products of inertia rather than the negative of the products of inertia.

The spacecraft inertia matrix, $\boldsymbol{I}_s$, is defined as the inertia matrix of the entire spacecraft treated as a single rigid body (i.e., with the rotor stationary relative to the platform). The ij component of $\boldsymbol{I}_s$ is denoted I_{ij}. As previously mentioned, the coordinate system can be selected so that $I_{12} \equiv I_{21} \equiv 0$. Thus,

$$\boldsymbol{I}_s = \begin{bmatrix} I_{11} & 0 & I_{13} \\ 0 & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} \tag{1}$$

With the exception of the 3-3 term, the platform inertia matrix $\boldsymbol{I}_p$ is identical to $\boldsymbol{I}_s$. In $\boldsymbol{I}_p$, the 3-3 term excludes the axial inertia of the wheel I_r. Hence,

$$\boldsymbol{I}_p = \begin{bmatrix} I_{11} & 0 & I_{13} \\ 0 & I_{22} & I_{23} \\ I_{13} & I_{23} & (I_{33}-I_r) \end{bmatrix} \tag{2}$$

The determinants of $\boldsymbol{I}_s$ and $\boldsymbol{I}_p$ are denoted by Δ_s and Δ_p, respectively.

The vector $\boldsymbol{h}_r$ is defined as the angular momentum of the rotor about its symmetry axis due to the combined effects of platform rotation plus rotation of the rotor relative to the platform. In mathematical notation,

$$\boldsymbol{h}_r = (\omega_3 + \omega_r) I_r \boldsymbol{e}_3 \tag{3}$$

where ω_3 is the angular rate of the platform about the 3 axis, ω_r the angular rate of the rotor relative to the platform, and $\boldsymbol{e}_3$ a unit vector in the 3 direction.

The vector $\boldsymbol{h}_p$ is the angular momentum of the platform plus the component of the rotor angular momentum that is perpendicular to the rotor axis. In other words, $\boldsymbol{h}_p$ is the difference between the total angular momentum ($\boldsymbol{h}_s$) and $\boldsymbol{h}_r$. From the definitions of θ and ϕ, it is easy to show that

$$\boldsymbol{h}_p = \begin{bmatrix} h_s\cos\phi\sin\theta \\ h_s\sin\phi\sin\theta \\ h_s\cos\theta - h_r \end{bmatrix} \tag{4}$$

where h_s and h_r are the magnitudes of $\boldsymbol{h}_s$ and $\boldsymbol{h}_r$.

Defining the normalized rotor momentum ψ as

$$\psi \equiv \omega_r I_r / h_s \tag{5}$$

and noting that

$$\omega_3 = \boldsymbol{e}_3^T \boldsymbol{I}_p^{-1} \boldsymbol{h}_p \tag{6}$$

it is seen that the magnitude of $\boldsymbol{h}_r$ may be written

$$h_r = I_r \boldsymbol{e}_3^T \boldsymbol{I}_p^{-1} \boldsymbol{h}_p + \psi h_s \tag{7}$$

Substitution of Eqs. (2) and (4) into Eq. (7) and solving for h_r yields

$$h_r = h_s \Delta_s^{-1} [I_{11} I_{22} I_r \cos\theta - I_r f(\phi)\sin\theta + \psi\Delta_p] \tag{8}$$

where

$$f(\phi) \equiv I_{11} I_{23} \sin\phi + I_{22} I_{13} \cos\phi \tag{9}$$

The total rotational kinetic energy of the spacecraft is

$$S = \tfrac{1}{2} \boldsymbol{h}_p^T \boldsymbol{I}_p^{-1} \boldsymbol{h}_p + \tfrac{1}{2} I_r^{-1} h_r^2 \tag{10}$$

As indicated in the introduction, however, the analysis will center on the core energy C, which is obtained by subtracting from S the energy due to motion of the rotor relative to the platform:

$$C = \tfrac{1}{2} \boldsymbol{h}_p^T \boldsymbol{I}_p^{-1} \boldsymbol{h}_p + \tfrac{1}{2} I_r^{-1} (h_r - \psi h_s)^2 \tag{11}$$

Substituting Eqs. (2), (4), and (8) into Eq. (11) yields, after some messy algebra,

$$C(\theta,\phi) = \tfrac{1}{2} h_s^2 (I_{11} I_{22})^{-1} \{ (I_{22}\cos^2\phi + I_{11}\sin^2\phi)\sin^2\theta + [I_{11} I_{22}(\psi - \cos\theta) + f(\phi)\sin\theta]^2 \Delta_s^{-1} \} \tag{12}$$

The aim of the analysis is to determine the criteria such that passive energy dissipation causes the rotor axis to become aligned with the total angular momentum vector. In other words, the object is to find the conditions under which θ will asymptotically approach zero.

If a passive motion damper is present on the platform, then, according to Ref. 5, $C(\theta,\phi)$ will tend toward a minimum. It is assumed, of course, that the rotor is maintained at a constant speed relative to the platform and that energy dissipation on the rotor is negligible. If the system parameters are chosen so that $C(\theta,\phi)$ has a minimum at $\theta = 0$, and the minimum is unique, the rotor axis will ultimately become aligned with the angular momentum vector, regardless of the initial conditions. The requirement for uniqueness is important. If $C(\theta,\phi)$ does not have a unique minimum, it is possible for the spacecraft to converge to an unwanted orientation. Undesirable minima can be described as "energy traps."

To determine the conditions under which $C(\theta,\phi)$ has a unique minimum at $\theta=0$, all extrema must be located and their natures determined. Toward this end, the first and second partial derivatives of $C(\theta,\phi)$ with respect to θ and ϕ are required. These partial derivatives (denoted C_θ, C_ϕ, $C_{\theta\theta}$, $C_{\phi\phi}$, and $C_{\theta\phi}$) are

$$C_\theta(\theta,\phi)=h_s^2(I_{11}I_{22})^{-1}\{[I_{11}I_{22}(\psi-\cos\theta)+f(\phi)\sin\theta]\cdot[I_{11}I_{22}\sin\theta+f(\phi)\cos\theta]\Delta_s^{-1}+(I_{22}\cos^2\phi+I_{11}\sin^2\phi)\sin\theta\cos\theta\} \tag{13}$$

$$C_\phi(\theta,\phi)=h_s^2(I_{11}I_{22})^{-1}\left\{(I_{11}-I_{22})\cos\phi\sin\phi\sin^2\theta+[I_{11}I_{22}(\psi-\cos\theta)+f(\phi)\sin\theta]\Delta_s^{-1}\sin\theta\frac{d}{d\phi}f(\phi)\right\} \tag{14}$$

$$C_{\theta\theta}(\theta,\phi)=h_s^2(I_{11}I_{22})^{-1}\{[I_{11}I_{22}\sin\theta+f(\phi)\cos\theta]^2\Delta_s^{-1}+[I_{11}I_{22}(\psi-\cos\theta)+f(\phi)\sin\theta]$$
$$\cdot[I_{11}I_{22}\cos\theta-f(\phi)\sin\theta]\Delta_s^{-1}+(I_{22}\cos^2\phi+I_{11}\sin^2\phi)(\cos^2\theta-\sin^2\theta)\} \tag{15}$$

$$C_{\phi\phi}(\theta,\phi)=h_s^2(I_{11}I_{22})^{-1}\left\{\left[\frac{d}{d\phi}f(\phi)\right]^2\Delta_s^{-1}\sin^2\theta-[I_{11}I_{22}(\psi-\cos\theta)+f(\phi)\sin\theta]\Delta_s^{-1}f(\phi)\sin\theta+(I_{11}-I_{22})(\cos^2\phi-\sin^2\phi)\sin^2\theta\right\} \tag{16}$$

and

$$C_{\theta\phi}(\theta,\phi)=h_s^2(I_{11}I_{22})^{-1}\left\{2\Delta_s^{-1}f(\phi)\frac{d}{d\phi}f(\phi)\sin\theta\cos\theta+I_{11}I_{22}\Delta_s^{-1}[(\psi-\cos\theta)\cos\theta+\sin^2\theta]\frac{d}{d\phi}f(\phi)+2(I_{11}-I_{22})\cos\phi\sin\phi\cos\theta\sin\theta\right\} \tag{17}$$

It is of interest to note that $C(\theta,\phi)$ and its partial derivatives are completely independent of I_r. The rotor appears only through the normalized rotor momentum ψ. Hence, the analytical results will not be a function of the physical size of the rotor. Only its momentum matters.

Attention will now be directed to the important special case in which the wheel axis is parallel to a principal axis. This case is of particular interest since most dual-spin satellites fall into this category. Defining the 3 axis to be a principal axis reduces Eqs. (13) and (14) to

$$C_\theta(\theta,\phi)$$
$$=h_s^2[I_{33}^{-1}(\psi-\cos\theta)\sin\theta+(I_{11}^{-1}\cos^2\phi+I_{22}^{-1}\sin^2\phi)\sin\theta\cos\theta] \tag{18}$$

$$C_\phi(\theta,\phi)=h_s^2(I_{22}^{-1}-I_{11}^{-1})\cos\phi\sin\phi\sin^2\theta \tag{19}$$

The spacecraft will be in dynamic equilibrium at those values of θ and ϕ that yield $C_\theta=C_\phi=0$. Assuming I_{11}, I_{22}, and I_{33} to have distinctly different values, there will be either two, four, or six equilibrium points, depending upon the value of ψ.

The equilibrium points which exist for all values of ψ are

$$\theta=0 \qquad \phi=\text{undefined} \tag{20}$$

and

$$\theta=180\text{ deg} \qquad \phi=\text{undefined} \tag{21}$$

Note that from the definitions of θ and ϕ, the value of ϕ is undefined at $\theta=0$ and $\theta=180$ deg. Hence, Eqs. (20) and (21) each represents a single point and not a family of points. The four remaining equilibrium points, if they exist, are located at

$$\theta=\cos^{-1}[\psi(1-I_{33}I_{11}^{-1})^{-1}] \qquad \phi=0 \text{ or } 180\text{ deg} \tag{22}$$

$$\theta=\cos^{-1}[\psi(1-I_{33}I_{22}^{-1})^{-1}] \qquad \phi=\pm 90\text{ deg} \tag{23}$$

A simple test of the results presented in Eqs. (20-23) may be made by considering the limiting case in which $\psi=0$ (i.e., simple rigid-body rotation). In that circumstance, the results indicate that rotation in either direction about each of the three principal axes represents a state of equilibrium. That, of course, is the correct solution.

As formerly mentioned, the object of the analysis is to determine the conditions under which $C(\theta,\phi)$ will have a unique minimum at $\theta=0$. This is done by using a Taylor expansion to examine the nature of each of the equilibrium points. The expansion about the point (θ_0,ϕ_0) is

$$C(\theta,\phi)=C(\theta_0,\phi_0)+(1!)^{-1}[(\theta-\theta_0)C_\theta(\theta_0,\phi_0)$$
$$+(\phi-\phi_0)C_\phi(\theta_0,\phi_0)]+(2!)^{-1}[(\theta-\theta_0)^2C_{\theta\theta}(\theta_0,\phi_0)$$
$$+(\phi-\phi_0)^2C_{\phi\phi}(\theta_0,\phi_0)+2(\theta-\theta_0)(\phi-\phi_0)C_{\theta\phi}(\theta_0,\phi_0)]$$
$$+\cdots \tag{24}$$

If (θ_0,ϕ_0) is an equilibrium point, the first order terms in the expansion are zero, and the nature of the equilibrium may be determined by examining the second order terms. If the sum of the second order terms is positive definite, then $C(\theta_0,\phi_0)$ is a minimum and the equilibrium is stable. A negative definite second order term indicates that $C(\theta_0,\phi_0)$ is a maximum and, therefore, that the equilibrium is unstable. If the sum of the second order terms has both positive and negative values in the vicinity of (θ_0,ϕ_0), then the point is a saddle. This is also an unstable equilibrium condition. The equilibrium point at $\theta=0$ will be investigated first.

Examination of Eqs.(16) and (17) reveals that, for the principal axis case, $C_{\theta\phi}(0,\phi)=C_{\phi\phi}(0,\phi)=0$. Hence, the second order term of the Taylor expansion will be positive definite at $\theta=0$ if and only if

$$C_{\theta\theta}(0,\phi)>0 \tag{25}$$

for all ϕ. This is, therefore, the requirement which must be met in order for $\theta=0$ to be a stable equilibrium point.

Substituting $I_{13}=I_{23}=0$ and $\theta=0$ into Eq. (15) and substituting the result into Eq. (25) yields, after some algebra,

$$\psi-1+I_{33}(I_{11}^{-1}\cos^2\phi+I_{22}^{-1}\sin^2\phi)>0 \tag{26}$$

This inequality will be satisfied for all values of ϕ if

$$\psi>1-I_{33}/I_{11} \tag{27}$$

and

$$\psi>1-I_{33}/I_{22} \tag{28}$$

As one might expect, inequalities Eqs. (27) and (28) are the well-known stability requirements for dual-spin spacecraft, although they may not be immediately recognizable as such due to our somewhat unusual notation. A little algebra, however, will readily transform these inequalities into more standard form

$$I_r\omega_r + (I_{33} - I_{11})\omega_3 > 0 \tag{29}$$

and

$$I_r\omega_r + (I_{33} - I_{22})\omega_3 > 0 \tag{30}$$

Having established the criteria for the core energy to have a minimum at $\theta = 0$, it is now necessary to determine the conditions for ensuring the uniqueness of that minimum. Since $\theta = 180$ deg is the only other equilibrium point which exists for all values of ψ, it is appropriate that it be examined next.

Since $C_{\phi\phi}(180\ \text{deg},\phi) = C_{\theta\phi}(180\ \text{deg},\phi) = 0$ when the 3 axis is a principal axis, a sufficient condition for $\theta = 180$ deg not to be a minimum is for at least one ϕ_0 to exist such that $C_{\theta\theta}(180\ \text{deg},\phi_0) < 0$. From Eq. (15) it is seen that this condition will be satisfied if

$$-\psi - 1 + I_{33}(I_{11}^{-1}\cos^2\phi_0 + I_{22}^{-1}\sin^2\phi_0) < 0 \tag{31}$$

has a solution. A solution will exist if either

$$\psi > -1 + I_{33}/I_{11} \tag{32}$$

or

$$\psi > -1 + I_{33}/I_{22} \tag{33}$$

or both are satisfied. If both inequalities are satisfied, then the second-order term of the Taylor series expansion about (180 deg,ϕ) is negative definite for all ϕ and $C(\theta,\phi)$ is a maximum at that point. Satisfying one, but not both, of the inequalities Eqs. (32) and (33) causes $\theta = 180$ deg to be a saddle point. In either case, $\theta = 180$ deg is unstable, which is the desired result.

A careful examination of inequalities Eqs. (27), (28), (32), and (33) and how they relate to Eqs. (22) and (23) leads to some very interesting and useful observations. First, consider the inequalities. For a simple spinning body ($\psi = 0$) exactly two of the four inequalities are satisfied. Which two of the four are satisfied depends upon the inertial properties. The important thing is that two, and only two, are satisfied.

As the wheel is spun up, ψ increases to where eventually all four inequalities are satisfied. Since I_{33} is greater than zero, and since I_{33} cannot be more than double both I_{11} and I_{22} (by the definition of the moments of inertia, this is a physical impossibility), at least three of the four inequalities must be satisfied before $\psi = 1$. Two of the satisfied inequalities will be Eqs. (27) and (28). Hence, $\theta = 0$ will be stable whereas $\theta = 180$ deg will be unstable for a ψ_0 somewhere in the range of $0 < \psi_0 \le 1$, and this will hold true for all ψ greater than ψ_0.

Now consider the equilibria located by Eqs. (22) and (23). For $\psi \ge 0$, if two of the four inequalities just discussed are unsatisfied, then both equations have solutions and there are a total of six equilibrium points. If only one of the four inequalities is unsatisfied, either Eqs. (22) or (23) has a solution, but not both. Hence, satisfying the third inequality is associated with the annihilation of a pair of equilibrium points (leaving four). If all four inequalities are satisfied, neither Eq. (22) nor (23) has a solution and the system is left with only two equilibrium points (at $\theta = 0$ and $\theta = 180$ deg). These results are summarized in Table 1. One is led to conclude, therefore, that the stabilization of $\theta = 0$ and the destabilization of $\theta = 180$ deg is accomplished by eliminating one or more pairs of undesirable equilibrium points. The observations of this paragraph will now be put to use.

Consider the situation in which the core energy has a minimum at $\theta = 0$ and a maximum at $\theta = 180$ deg (i.e., Eqs. (27), (28), (32), and (33) are all satisfied). For this case, as just discussed, $\theta = 0$ and $\theta = 180$ deg are the only equilibrium points and the minimum is unique. This, of course, is the desired result.

The next situation to be considered is that in which the core energy has a minimum at $\theta = 0$ and a saddle at $\theta = 180$ deg [i.e., Eqs. (27) and (28) are both satisfied and either Eq. (32) or (33) are satisfied, but not both]. With one of the four inequalities unsatisfied, four equilibrium points exist. In addition to the equilibria at $\theta = 0$ and $\theta = 180$ deg, a pair of equilibrium points is given by either Eq. (22) or (23). These two additional equilibrium points have identical values of θ and have values of ϕ that differ by exactly 180 deg. Examination of Eq. (12) shows that whether these points are located by Eq. (22) or (23), they represent identical energies. These equilibria, therefore, are energy maxima since at least one maximum must exist and neither $\theta = 0$ nor $\theta = 180$ deg is a maximum. Hence, the minimum at $\theta = 0$ is unique when $\theta = 180$ deg is a saddle point.

It has thus been demonstrated that the core energy has a unique minimum at $\theta = 0$ if inequalities Eqs. (27) and (28) are both satisfied *and* either Eq. (32) or (33), or both, are satisfied. This requirement for uniqueness can be reduced to the single inequality

$$\psi > |1 - I_{33}/I_m| \tag{34}$$

where I_m is defined as being equal to either I_{11} or I_{22}, whichever is greater.

With a sufficiently high rotor speed, inequality Eq. (34) can always be satisfied. Hence, regardless of the initial conditions, a platform-mounted energy dissipation mechanism can be used to cause the rotor axis to become aligned parallel to the total angular momentum vector.

Attention is now directed to the case in which the spaecraft 3 axis is not a principal axis. This case is not as often found in practice and is included primarily for completeness.

Table 1 Relationship between ψ and the number of equilibria

Normalized wheel momentum ψ	Number of satisfied inequalities[a]	Number of equilibrium points	Location of the equilibrium points
$0 \le \psi < \|1 - I_{33}/I_{11}\|$ and $0 \le \psi < \|1 - I_{33}/I_{22}\|$	2	6	$\theta = 0$, $\theta = 180$ deg, two given by Eq. (22), and two given by Eq. (23)
$\|1 - I_{33}/I_{11}\| < \psi < \|1 - I_{33}/I_{22}\|$ or $\|1 - I_{33}/I_{22}\| < \psi < \|1 - I_{33}/I_{11}\|$	3	4	$\theta = 0$, $\theta = 180$ deg, and two given by either Eq. (22) or (23)
$\psi > \|1 - I_{33}/I_{11}\|$ and $\psi > \|1 - I_{33}/I_{22}\|$	4	2	$\theta = 0$ and $\theta = 180$ deg

[a] Of inequalities Eqs. (27), (28), (32), and (33).

Investigation of Eqs. (13) and (14) shows that $C(\theta,\phi)$ has a stationary point at $\theta=0$ if and only if

$$\psi = 1 \tag{35}$$

From Eqs. (16) and (17), it is easy to show that $C_{\theta\phi}(0,\phi) = C_{\phi\phi}(0,\phi) = 0$ for all ϕ. Hence, as for the principal axis case, the stationary point will be a minimum if

$$C_{\theta\theta}(0,\phi) > 0 \tag{36}$$

for all ϕ. Upon examining Eq. (15), it is seen that, with $\psi=1$, this requirement will be met if

$$\Delta_s^{-1} f^2(\phi) + (I_{22}\cos^2\phi + I_{11}\sin^2\phi) > 0 \tag{37}$$

where $f(\phi)$ is as defined in Eq. (9). Since all terms to the left of the inequality are greater than or equal to zero, and since they cannot all be zero simultaneously, inequality Eq. (37) is satisfied for all ϕ. Hence, the core energy has a minimum at $\theta=0$ when $\psi=1$.

A brief examination of Eq. (12) reveals that $C(0,\phi)=0$ when $\psi=1$. This, of course, is what one would expect since the body of the spacecraft is stationary when $\theta=0$ with $\psi=1$. Further examination of Eq. (12) shows that $C(\theta,\phi)>0$ when $\theta\neq0$. Hence, when $\psi=1$, $\theta=0$ is not merely a minimum, it is an absolute minimum.

As of this writing, the uniqueness of the minimum at $\theta=0$ has not been established conclusively for the nonprincipal axis case. No counter examples have been found, however, and the author can find no reason to doubt the uniqueness of the minimum when $\psi=1$.

Computer Simulations

Numerous computer simulations have been performed to verify the preceding analysis. The results of some of these simulations will now be presented in order to illustrate two interesting and potentially useful maneuvers.

For the purpose of the simulations, it was necessary to add a platform-mounted energy dissipation device to the mathematical model. In the computer program that produced the results presented here, the dissipator is modeled as a wheel that is attached to the platform and that has its axis of rotation parallel to the 1 axis. The torque about this axis is proportional to the relative rate of rotation between the damper wheel and the platform (i.e., the dissipation mechanism is viscous damping). This particular dissipation mechanism was chosen for its mathematical simplicity and because its action is analogous to that of the often used fluid-filled ring damper. It should be noted, however, that the energy dissipation rate was chosen to be somewhat higher than what one would normally find on a spacecraft. This was done to compress the time required for the various maneuvers in order that the results might be more vividly demonstrated.

For all the simulations presented here, the rotor axis is aligned parallel to the spacecraft's maximum moment of inertia axis. The inertias were chosen to be $I_{11}=91$ kg-m^2, $I_{22}=100$ kg-m^2, and $I_{33}=105$ kg-m^2. While these inertias do not represent any particular spacecraft, they can be considered fairly typical for a satellite in the 400-500-kg class.

With the above inertias, the point $\theta=0$ deg is an energy minimum even when $\psi=0$. Hence, it is stable even without the aid of the wheel. This minimum, however, is not unique. Another exists at $\theta=180$ deg. Thus, as far as attitude acquisition is concerned, the primary purpose of the wheel is to eliminate the ambiguity. From inequalities Eqs. (32) and (33), we see that this requires ψ to be greater than 0.05.

In simulating the first maneuver, the spacecraft was placed in an initial state of simple spin about its minor axis ($\theta=90$ deg, $\psi=0$). The rotor was then spun up by a constant motor torque until $\psi=0.09$. Once rotor spin up was complete (after about 84 s), the rotor was maintained at a constant rate relative to the platform. The resulting nutation angle history is shown in Fig. 2 and the histories of the core energy and the total energy appear in Fig. 3.

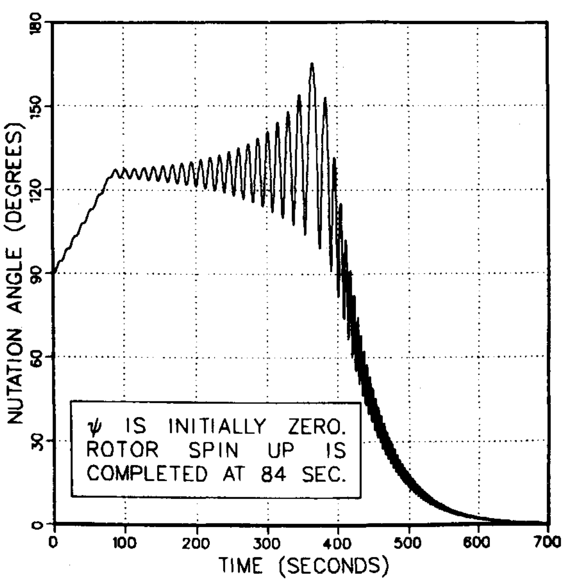

Fig. 2 Nutation angle vs time during a minor-to-major axis turn with $\psi=0.09$.

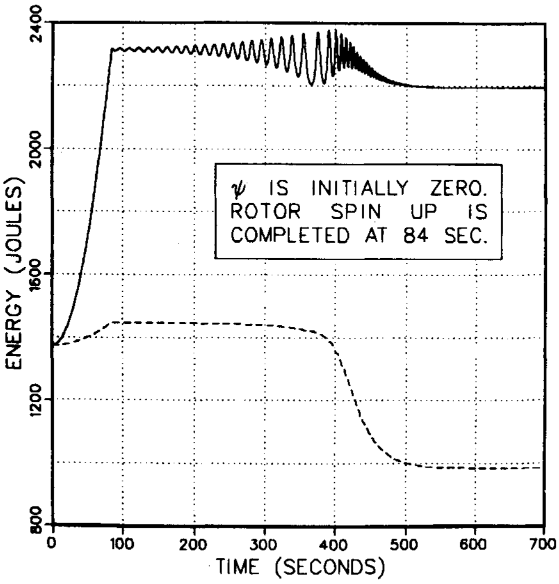

Fig. 3 Core energy (dashed line) and total energy (solid line) vs time during a minor-to-major axis turn with $\psi=0.09$.

Examination of Fig. 2 shows that, for a maneuver of this type, reaction torques on the platform during rotor spin up cause the nutation angle to grow rather than to decrease. This divergence from the desired end condition ($\theta=0$ deg) has led to the rejection of such maneuvers as being unfeasible.[4] As Fig. 2 illustrates, however, the initial inversion is merely an inconvenience and does not represent a barrier. Once the rotor rate has been brought up to a level such that ψ satisfies inequality Eq. (32) or (33), energy dissipation causes convergence to the correct attitude.

For this particular case, the normalized rotor momentum of 0.09 causes $\theta=180$ deg to be a saddle point and the energy maxima are located at $\theta=125.8$ deg, $\phi=0$ deg and at $\theta=125.8$ deg, $\phi=180$ deg. Note that following wheel spin up, the motion is initially characterized by a divergent oscillation about the first of these maxima. There is excellent agreement between the simulation and the analysis as to the location of this point.

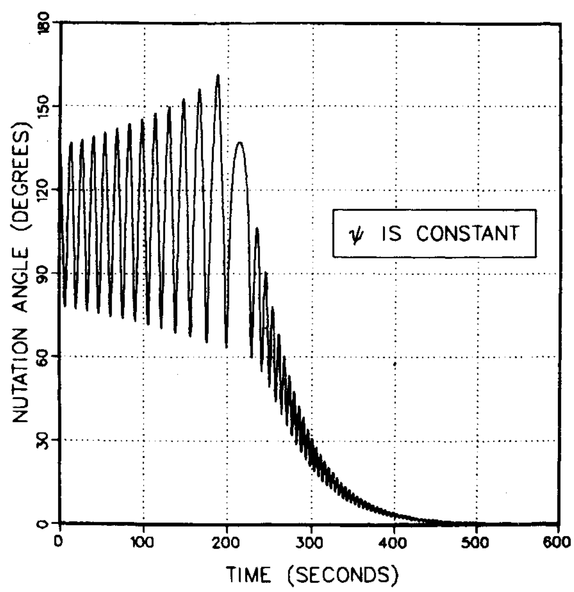

Fig. 4 Nutation angle vs time with $\psi=0.04$ (convergence to the correct attitude).

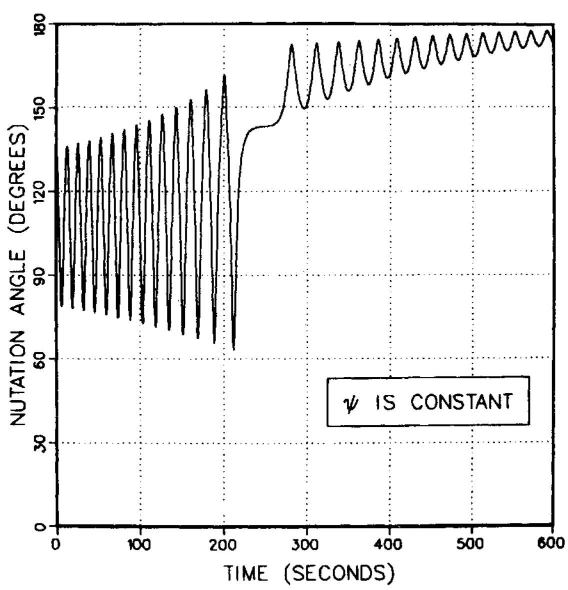

Fig. 6 Nutation angle vs time with $\psi=0.04$ (convergence to an inverted attitude).

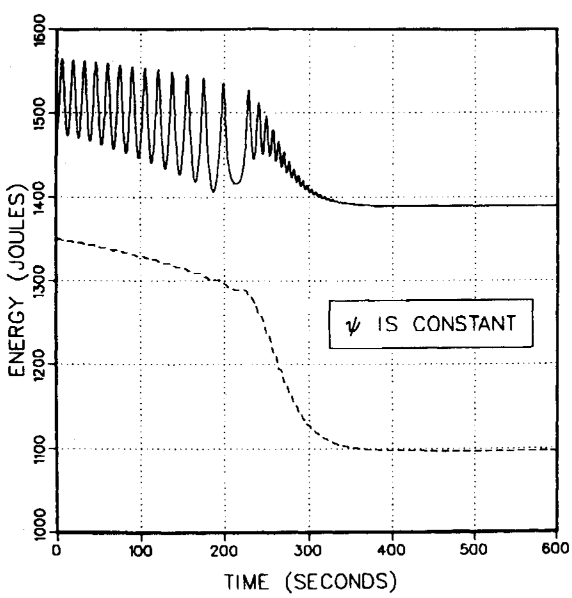

Fig. 5 Core energy (dashed line) and total energy (solid line) vs time with $\psi=0.04$ (convergence to the correct attitude).

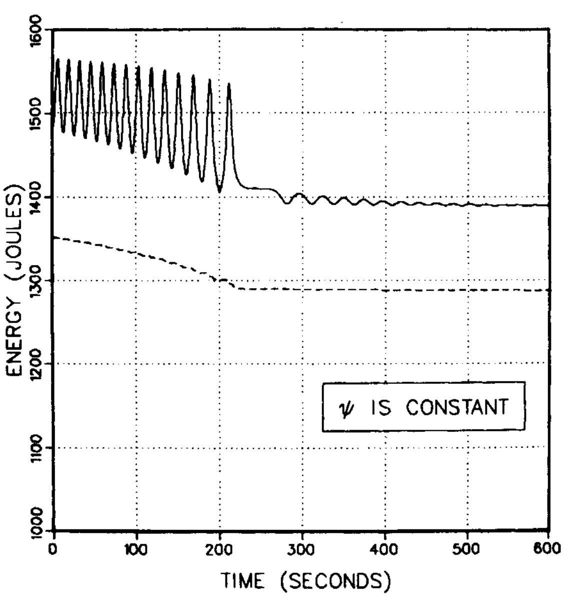

Fig. 7 Core energy (dashed line) and total energy (solid line) vs time with $\psi=0.04$ (convergence to an inverted attitude).

The second and third simulations (shown in Figs. 4-7) illustrate the ambiguity that exists if the normalized rotor momentum is less than 0.05. The two simulations were run with identical rotor speeds ($\psi=0.04$) but slightly different initial conditions. In both cases, however, the rotor is already up to speed at time zero. For the first 200 s, the motions are quite similar, being characterized by nutation angle oscillations about $\theta=105.1$ deg. Shortly thereafter, however, there is a significant divergence as the first case decays to 0 deg nutation angle which the second asymptotically approaches 180 deg. This is a graphic illustration of the fact that stability of $\theta=0$ deg is not a sufficient condition to assure convergence to that point. Destabilization of $\theta=180$ deg is also required.

The fourth simulation is an example of how rotor spinup can be used to creatively destabilize an initially stable motion. In this case, the spacecraft is initially in a state of simple spin about the maximum moment of inertia axis ($\theta=180$ deg, $\psi=0$) and the object is to reverse the direction of the spin. The results are shown in Figs. 8 and 9. The procedure is to torque the rotor so that it acquires a spin opposite to that of the platform. When the rotor speed exceeds the rate required for $\psi=0.05$, the initial orientation becomes unstable and an inversion occurs. For the results shown, the rotor was spunup until $\psi=0.09$. Destabilization ($\psi=0.05$) occurs at the 50-s mark and spinup is complete at 92 s. It should be noted that the initial conditions included a very small nutational motion in order to assure that divergence from $\theta=180$ deg would occur. This represents a realistic initial condition since, in practice, small attitude perturbations are always present.

Although it is not shown here, convergence to $\theta=0$ deg could be followed by despin of the rotor. This would leave the spacecraft, once again, in a state of simple spin about its maximum moment of inertia axis but with the direction of the spin reversed from the initial condition.

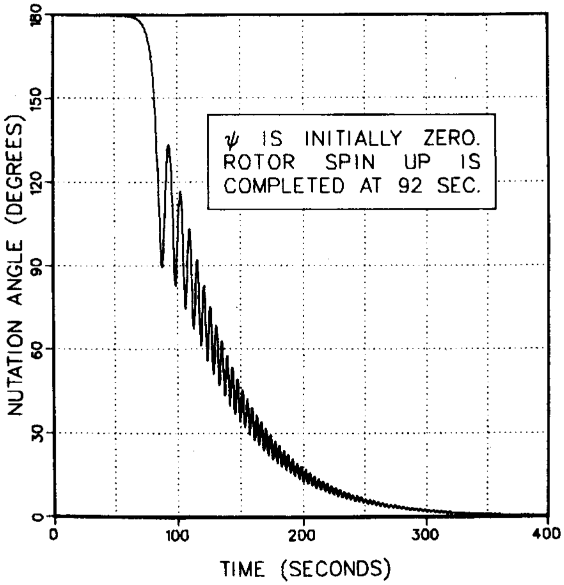

Fig. 8 Nutation angle vs time with $\psi = 0.09$ (reversal of spin about the major axis).

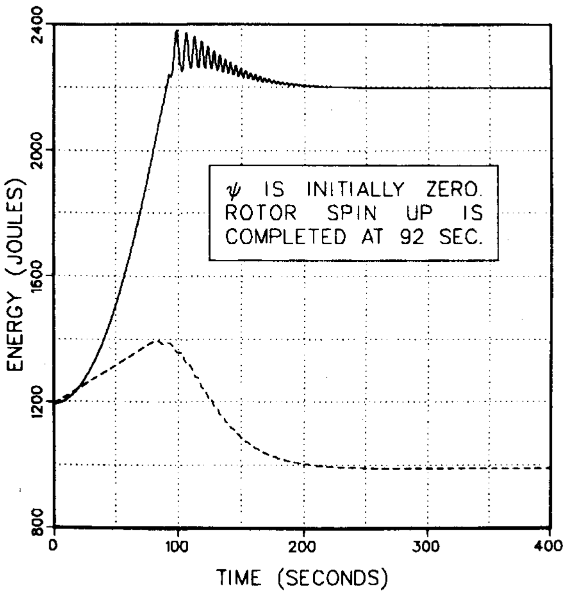

Fig. 9 Core energy (dashed line) and total energy (solid line) vs time with $\psi = 0.09$ (reversal of spin about the major axis).

Conclusion

It has been demonstrated analytically, and confirmed by simulation, that rotor spinup followed by passive energy dissipation can be used to orient the rotor axis of a dual-spin spacecraft parallel to the total angular momentum vector. With the proper choice of rotor speed, the initial conditions and dynamic history of the spacecraft during rotor spinup are irrelevant to the ultimate convergence of the rotor axis to the desired orientation. Because of this, a number of maneuvers previously thought to be impossible are now known to be feasible. A specific maneuver of this type is the acquisition of a stable momentum bias about either the axis of maximum or intermediate moment of inertia when the spacecraft is initially spinning about the axis of minimum moment of inertia. It has been shown that the initial divergence reported by Kaplan[4] does not represent a barrier to successful completion of this maneuver. The possibility of inverting a spacecraft relative to its angular momentum vector has also been demonstrated.

The basis of the attitude acquisition technique is the creation of a unique minimum in the core energy. It should be recalled, however, that the analysis is based on the assumption that the energy of the damper is negligible. If the mass and motions of the dissipation device are sufficiently large, additional nontrivial energy minima can be introduced. This presents the possibility that the spacecraft can become trapped in an undesired orientation. The convergence requirements for such a system may be different (and more severe) than those presented here.

In deciding whether to employ the attitude acquisition method presented in this paper, the considerations of the preceding paragraph should be kept in mind. Nevertheless, where applicable, this technique presents several advantages. Specifically, the only active system required is a rotor speed control, there is no need for attitude determination during the maneuver, and no expendables are used.

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